Guts Round

Lexington High School

December 8, 2018

8th Annual Lexington Math Tournament - Guts Round - Part 1

Team Name: _____

_____ 1. **[5]** Evaluate the sum $1-2+3\cdots-208+209-210$.

_____ 2. **[5]** Tony has 14 beige socks, 15 blue socks, 6 brown socks, 8 blond socks and 7 black socks. If Tony picks socks out randomly, how many socks does he have to pick in order to guarantee a pair of blue socks?

_____ 3. **[5]** The price of an item is increased by 25%, followed by an additional increase of 20%. What is the overall percentage increase?

8th Annual Lexington Math Tournament - Guts Round - Part 2

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_____ 4. **[5]** A lamp post is 20 feet high. How many feet away from the base of the post should a person who is 5 feet tall stand in order to cast an 8-foot shadow?

_____ 5. **[5]** How many positive even two-digit integers are there that do not contain the digits 0, 1, 2, 3 or 4?

_____ 6. **[5]** In four years, Jack will be twice as old as Jill. Three years ago, Jack was three times as old as Jill. How old is Jack?

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_____ 7. **[6]** Let $x\Delta y = xy^2 - 2y$. Compute $20\Delta 18$.

_____ 8. **[6]** A spider crawls 14 feet up a wall. If Cheenu is standing 6 feet from the wall, and is 6 feet tall, how far must the spider jump to land on his head?

_____ 9. **[6]** There are fourteen dogs with long nails and twenty dogs with long fur. If there are thirty dogs in total, and three do not have long fur or long nails, how many dogs have both long hair and long nails?

8th Annual Lexington Math Tournament - Guts Round - Part 4

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_____ 10. **[6]** Exactly 420 non-overlapping square tiles, each 1 inch by 1 inch, tesselate a rectangle. What is the least possible number of inches in the perimeter of the rectangle?

_____ 11. **[6]** John drives 100 miles at fifty miles per hour to see a cat. After he discovers that there was no cat, he drives back at a speed of twenty miles per hour. What was John's average speed in the round trip?

_____ 12. **[6]** What percent of the numbers $1, 2, 3, \ldots 1000$ are divisible by exactly one of the numbers 4 and 5?

8th Annual Lexington Math Tournament - Guts Round - Part 5

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_____ 13. **[7]** Express the number 3024_8 in base 2.

_____ 14. **[7]** $\triangle ABC$ has a perimeter of 10 and has $AB = 3$ and $\angle C$ has a measure of 60°. What is the maximum area of the triangle?

_____ 15. **[7]** A weighted coin comes up as heads 30% of the time and tails 70% of the time. If I flip the coin 25 times, how many tails am I expected to flip?

8th Annual Lexington Math Tournament - Guts Round - Part 6

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_____ 16. **[7]** A rectangular box with side lengths 7, 11, and 13 is lined with reflective mirrors, and has edges aligned with the coordinate axes. A laser is shot from a corner of the box in the direction of the line $x = y = z$. Find the distance traveled by the laser before hitting a corner of the box.

_____ 17. **[7]** The largest solution to $x^2 + \frac{49}{x^2} = 2018$ can be represented in the form $\sqrt{a} + \sqrt{b}$. Compute $a + b$.

_____ 18. **[7]** What is the expected number of black cards between the two jokers of a 54 card deck?

8th Annual Lexington Math Tournament - Guts Round - Part 7

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_____ 19. **[8]** Compute $\binom{6}{0} \cdot 2^0 + \binom{6}{1} \cdot 2^1 + \binom{6}{2} \cdot 2^2 + \cdots + \binom{6}{6} \cdot 2^6$

_____ 20. **[8]** Define a sequence by $a_1 = 5$, $a_{n+1} = a_n + 4 * n - 1$ for $n \geq 1$. What is the value of a_{1000}?

_____ 21. **[8]** Let $\triangle ABC$ be the triangle such that $\angle B = 15°$ and $\angle C = 30°$. Let D be the point such that $\triangle ADC$ is an isosceles right triangle where D is in the opposite side from A respect to BC and $\angle DAC = 90°$. Find the $\angle ADB$.

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_____ 22. **[8]** Say the answer to problem 24 is z. Compute

$$\gcd(z, 7z + 24)$$

_____ 23. **[8]** Say the answer to problem 22 is x. If x is 1, write down 1 for this question. Otherwise, compute

$$\sum_{k=1}^{\infty} \frac{1}{x^k}$$

_____ 24. **[8]** Say the answer to problem 23 is y. Compute

$$\left\lfloor \frac{y^2 + 1}{y} \right\rfloor$$

8th Annual Lexington Math Tournament - Guts Round - Part 9

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_____ 25. **[9]** A positive integer is called *spicy* if it is divisible by the sum if its digits. Find the number of spicy integers between 100 and 200 inclusive.

_____ 26. **[9]** Rectangle $ABCD$ has points E and F on sides AB and BC, respectively. Given that $\frac{AE}{BE} = \frac{BF}{FC} = \frac{1}{2}$, $\angle ADE = 30°$, and $[DEF] = 25$, find the area of rectangle $ABCD$.

_____ 27. **[9]** Find the largest value of n for which 3^n divides $\binom{100}{33}$.

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_____ 28. **[11]** Isosceles trapezoid $ABCD$ is inscribed in a circle such that $AB \parallel CD$, $AB = 2$, $CD = 4$, and $AC = 9$. What is the radius of the circle?

_____ 29. **[11]** Find the product of all possible positive integers n less than 11 such that in a group of n people, it is possible for every person to be friends with exactly 3 other people within the group. Assume that friendship is a mutual relationship.

_____ 30. **[11]** Compute the infinite product

$$\left(1+\frac{1}{2^1}\right)\left(1+\frac{1}{2^2}\right)\left(1+\frac{1}{2^4}\right)\left(1+\frac{1}{2^8}\right)\left(1+\frac{1}{2^{16}}\right)\cdots$$

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_____ 31. **[13]** Find the sum of all possible values of xy if $x+\frac{1}{y}=12$ and $\frac{1}{x}+y=8$.

_____ 32. **[13]** Find the number of ordered pairs (a,b), where $0<a,b<1999$, that satisfy

$$a^2+b^2 \equiv ab \pmod{1999}$$

_____ 33. **[13]** Let $f:\mathbb{N}\to\mathbb{Q}$ be a function such that $f(1)=0, f(2)=1$ and $f(n)=\frac{f(n-1)+f(n-2)}{2}$. Evaluate

$$\lim_{n\to\infty} f(n).$$

8th Annual Lexington Math Tournament - Guts Round - Part 12

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_____ 34. **[15]** Estimate the sum of the digits of 2018^{2018}. The number of points you will receive is calculated using the formula $\max(0, 15-\log_{10}(A-E))$, where A is the true value and E is your estimate.

_____ 35. **[15]** Let $C(m,n)$ denote the number of ways to tile an m by n rectangle with 1x2 tiles. Estimate $\log_{10}(C(100,2))$. The number of points you will recieve is calculated using the formula $\max(0, 15-\log_{10}(A-E))$, where A is the true value and E is your estimate.

_____ 36. **[15]** Estimate $\log_2\binom{1000}{500}$. The number of points you earn is equal to $\max(0, 15-|A-E|)$, where A is the true value and E is your estimate.